The Extension to Cook-Seiford Social Choice Function and the Choice of Key Information Technologies in Guangdong Province of China*

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Abstract- The usual Social Choice Functions deal with problems of group decision- making with single criterion. In this paper the Cook-Seiford Social Choice Function (CSSCF) is extended in two ways to process group decision-making problems with multiple criteria. Then, by using the function, a choice-analysis is made to key technologies for IT industries of Guangdong province, China, in the years 2005 – 2010.

Key words: Cook-Seiford Social Choice Function; multi-criteria decision-making technology; technology choice of IT industry

I. INTRODUCTION

According to K. O. May's definition, a Social Choice Function should have follow The multi-people decision-making problem, also called Group Decision problem, is plenty in social issues. For example, voting is one of the most commonly used Group Decision Methods. However, because the so-called Paradox of Voting is inevitable, people try to apply various quantitative indexes to figure out the group (social) preferences. These quantitative indexes are called as Social Choice Functions. Different Social Choice Functions have different quantitative indexes accordingly. They reflect different principles of group members' preferences.

ing features: decisiveness, neutrality, anonymity, monotonicity, unanimity and Pareto optimality. But not all Social Choice Functions have all these features; generally they only have some of them. The common Social Choice Functions are Condorcet Function、Borda Function、Nanson Function, etc.

In human society, there are also plenty of multi-criteria decision problems and decision technologies for solving them, such as AHP, TOPSIS, ELECTRE, etc.

In fact, most decision problems are those of both multi-people and multi-criteria in nature. Here, on the basis of CSSCF, we study two multi-people decision methods and their applications as well.

II. COOK-SEIFORD SOCIAL CHOICE FUNCTION (CSSCF)

According to Cook-Seiford, a ranking to certain alternatives by the group should match as close as possible to the ranking by group members. A distance function is introduced to evaluate the disagreement between the two rankings. So, ranking of the group should make the general distance to the minimum. The mathematic expression is as follows:

$$
\begin{aligned}
&Min \quad z = \sum_{j=1}^{m} \sum_{k=1}^{m} d_{jk} p_{ik} \\
&s.t. \quad \sum_{j=1}^{m} p_{jk} = 1 \qquad (1) \\
&\qquad \sum_{k=1}^{m} p_{jk} = 1 \qquad (j, k = 1, \ 2 \ \ldots \ m) \\
&p_{jk} = 0 \ , \ 1
\end{aligned}
$$

in which d_{jk} denotes the distance between rankings of the group and group members to alternative j in the case when the group ranks alternative j as k-th place. For instance, if 6 people rank alternative1 as the second place and 3 people rank it as the first place, then

$$d_{11} = 6|2-1| + 3|1-1| = 6$$

Here p_{jk} is decision variable, $p_{jk}=1$ denotes that the group rank alternative j as k place, $p_{jk}=0$ denotes that the group does not rank alternative j as k place. CSSCF has decisiveness, neutrality, anonymity, monotonicity, and objectivity but does not have Pareto optimality.

From the above-described definition of CSSCF we know this function does not directly deal with multi-criteria group decision problems. In other words, it demands that the group members first make the single-criterion synthetic ranking to the alternatives and then solve only the integrating problem of multi-people decision. However, as to the problem scene with greater complexity, people usually can only make single-criterion ranking to the alternatives, so results of group choice solved with CSSCF must be that under single-criterion.

III. THE EXTENSION TO CSSCF

In order to solve the multi-criteria group decision problem using CSSCF, in this paper we develop two methods to extend the function.

A. Weighted distance method

Comparing with the method in model (1), there is only a different definition of d_{jk} in our weighted distance method.

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Here d_{jk} is defined as weighted average value of the original d_{jk}s under single-criterion, and this is why this method has its name.

In this method the weight is equal to importance degree of the criterion accordingly.

To solve d_{jk} in the weighted distance method, we first use AHP or other methods to obtain the importance weight of every criterion, and then using this weight value to make revision to the original d_{jk}.

Suppose we have l criteria, the weight value of criterion r is w_r, and, under criterion r, d^r_{jk} is the distance between rankings of the group and group members to alternative j in the case when the group ranks alternative j as k-th place. Then in the weighted distance method,

$$d_{ik} = \sum_{r=1}^{l} w_r d^r_{jk} \qquad (2)$$

B. Cook-Seiford-Borda-AHP method

The idea in weighed distance method is that we first revise the definition of distance in CSSCF to solve the multi-criteria choice problem, and then use this function to solve the problem of multi-people choice. According to the symmetric thinking style, we can also first use CSSCF to solve the problem of multi-people choice, and then solve the multi-criteria choice problem with AHP and other methods. If we choose AHP, then the difficulty is in that the solution of CSSCF is only a ranking to the alternatives and we do not have the relative importance weights that are necessary for using AHP. To overcome this difficulty, we normalize Borta values of the alternatives and use them as the relative importance weights, then synthesize them to obtain the absolute weights and finally reach the results of multi-criteria group choices. Based on the sequence of using, we name this method as Cook-Seiford-Borda-AHP method.

In the weighted distance method, d_{jk} is linearly revised by importance weights and introduced into model (1); on the other hand, in Cook-Seiford-Borda-AHP method the CSSCF is used step by step, so both the methods do not change the features of CSSCF. We will discuss this question in another paper.

IV. THE APPLICATION IN CHOOSING PRIORITIZED TECHNOLOGIES FOR IT INDUSTRY OF GUANGDONG PROVINCE IN 2005 – 2010

In order to choose prioritized technologies for IT industry of Guangdong Province in 2005 – 2010, we visited 9 specialists in IT industry. First the researchers and specialists worked together to define the criteria, sub criteria and alternative technologies and then, produced the correspondent AHP hierarchical structure (Fig.1). Afterwards, the choice was made with the above-described two methods. The 9 alternative technologies are: ① IC technology ② computer and web product technology ③ modern communication technology ④ software ⑤ digital audio and video ⑥ basic electronic ⑦ new-type components and key materials ⑧ mechatronics based on automatic control and flexible manufacture and CAD ⑨ info-consumer electric device.

A. Steps and results of solution with the Weighted Distance method

1) We produce AHP judge matrices via investigation with the specialists, then the weight of every sub criterion is calculated as follows.

   (w_r) = (0.066, 0.332, 0.332, 0.027, 0.081, 0.081, 0.071, 0.010）

   The consistency test to all judgment matrices, including that of criteria level to goal level and every sub criterion to its criterion, has been passed in the solving process.
2) Nine specialists are invited to make the rankings to the alternatives under 8 sub criteria. (Original data of this part are omitted for saving the paper volume)
3) The distance matrices under 8 sub criteria are calculated respectively.
4) Weighted distance matrices （d_{jk}）are calculated on the basis of formula (2).
5) Rankings to the alternative technologies is produced using model (1). In fact model (1) is an assignment problem. With computer software we obtain the minimum distance 85.8 and the ranking result is

① $>_G$ ④ $_G$ ③ $>_G$ ② $>_G$ ⑤ $>_G$ ⑥ $>_G$ ⑧ $>_G$ ⑦ $>_G$ ⑨

B. Steps and results of the solution with Cook-Seiford-Borda-AHP method

1) With CSSCF we obtain the rank of alternative technologies under every criterion.

① $>_G$ ④ $>_G$ ③ $>_G$ ② $>_G$ ⑤ $>_G$ ⑥ $>_G$ ⑧ $>_G$ ⑦ $>_G$ ⑨

① $>_G$ ④ $>_G$ ③ $>_G$ ② $>_G$ ⑤ $>_G$ ⑥ $>_G$ ⑦ $>_G$ ⑨ $>_G$ ⑧

① $>_G$ ④ $>_G$ ③ $>_G$ ② $>_G$ ⑥ $>_G$ ⑧ $>_G$ ⑤ $>_G$ ⑦ $>_G$ ⑨

② $>_G$ ④ $>_G$ ① $>_G$ ③ $>_G$ ⑤ $>_G$ ⑦ $>_G$ ⑧ $>_G$ ⑨ $>_G$ ⑥

②>G ①>G ④>G ③>G ⑥ >G ⑦ >G ⑧ >G ⑤ >G ⑨

①>G ④>G ②>G ⑤>G ⑨ >G ⑥ >G ⑧ >G ③ >G ⑦

⑨>G ①>G ④>G ③>G ② >G ⑦ >G ⑧ >G ⑤ >G ⑥

③>G ①>G ④>G ②>G ⑤ >G ⑥ >G ⑧ >G ⑨ >G ⑦

The minimum total distances of corresponding assignment problem are as follows accordingly: 42, 54, 62, 114, 78, 90, 42, 11.

2）On the basis of the rank of alternative technologies, we produce the Borda value under every sub criterion, after normalization we have matrix B.

Criteria (columns); Alternative technologies (rows)

8	8	8	6	7	8	7	7
5	5	5	8	8	6	4	5
6	6	6	5	5	1	5	8
7	7	7	7	6	7	6	6
4	4	2	4	1	5	1	4
3	3	4	0	4	3	0	3
1	2	1	3	3	0	3	0
2	0	3	2	2	2	2	2
0	1	0	1	0	4	8	1

$$\mathbf{B} = \begin{vmatrix} 0.22 & 0.22 & 0.22 & 0.17 & 0.19 & 0.22 & 0.19 & 0.19 \\ 0.14 & 0.14 & 0.14 & 0.22 & 0.22 & 0.17 & 0.11 & 0.14 \\ 0.17 & 0.17 & 0.17 & 0.14 & 0.14 & 0.03 & 0.14 & 0.22 \\ 0.19 & 0.19 & 0.19 & 0.19 & 0.17 & 0.19 & 0.17 & 0.17 \\ 0.11 & 0.11 & 0.06 & 0.11 & 0.03 & 0.14 & 0.03 & 0.11 \\ 0.08 & 0.08 & 0.11 & 0.00 & 0.11 & 0.08 & 0.00 & 0.08 \\ 0.03 & 0.06 & 0.03 & 0.08 & 0.08 & 0.00 & 0.08 & 0.00 \\ 0.06 & 0.00 & 0.08 & 0.06 & 0.06 & 0.06 & 0.06 & 0.06 \\ 0.00 & 0.03 & 0.00 & 0.03 & 0.00 & 0.11 & 0.22 & 0.03 \end{vmatrix}$$

3) Using AHP we get the absolute weight A of alternative technologies under multi-criteria and consequently the group ranking result.

$$A = B \ (w_r)^T$$

$$= (0.215484 \quad 0.148167 \quad 0.151000 \quad 0.189944 \quad 0.082250 \quad 0.086639 \; 0.044417 \quad 0.046333 \quad 0.035078)^T$$

i.e. the ranking result to the alternative technologies is ①>G ④ >G ③ >G ② >G ⑥ >G ⑤ >G ⑧ >G ⑦ >G ⑨

C. Discussions

1) Analysis to the results

The calculation results by the two methods mentioned above are

①>G ④ >G ③ >G ② >G ⑤ >G ⑥ >G ⑧ >G ⑦ >G ⑨

①>G ④ >G ③ >G ② >G ⑥ >G ⑤ >G ⑧ >G ⑦ >G ⑨

We can see results by the two methods are almost the same with the exception that the ranking positions of alternative technology 5 and 6 are different. However, it is a question for further studies that whether or not there is inevitability in their similarity. Our first step analysis is that both of them use AHP and CSSCF. In Weighted distance method, AHP is first utilized to obtain weights under multi-criteria and the distance matrices are synthesized with these weights, then the multi-people decision problem is solved. But in the Cook-Seiford-Borda-AHP method first solved is the multi-people decision problem under every criterion, then the multi-criteria problem is solved with AHP.

However, in the latter method if we do not use Borta method to get the relative weights of the alternative technologies under every criterion, the results will be different.

2) The third method: AHP-CSSCF method

There is another method - first solve the multi-criteria and then multi-people decision problems. That is the AHP-CSSCF method. Being different from the weighted distance method, in this method AHP must be used many times to make multi-criteria ranking to the alternatives by every member of the group, then the CSSCF is used to synthesize the individual opinions and finally reach the result of group choice. The calculation in this method is enormous and, strictly speaking, it could not be considered as an extension to CSSCF. So in this paper we have not use it to make the study to the instances.

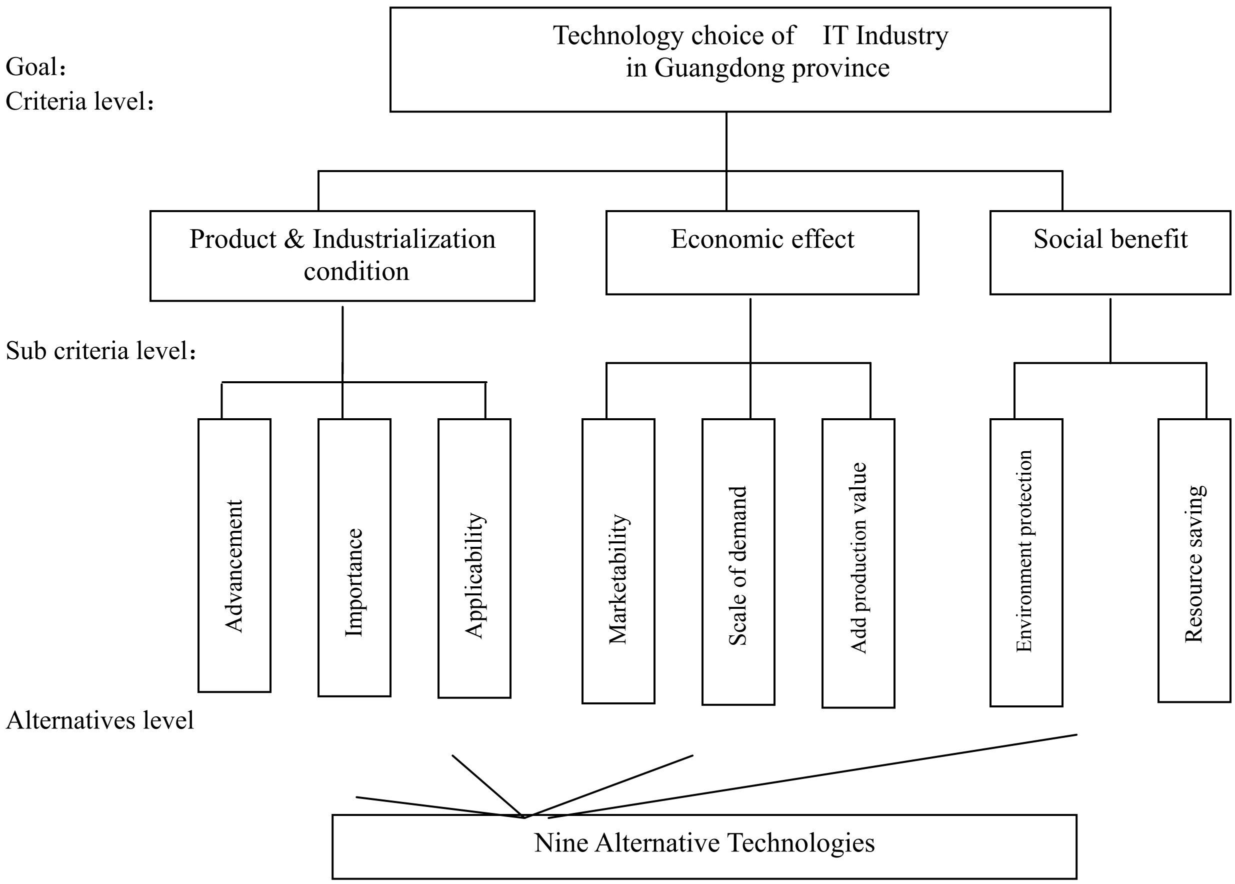

Fig.1 Hierarchy for the choice of Information technologies in Guangdong

V. CONCLUSIONS

In this paper, in order to deal with multi-criterion group decision problems, we use two methods, in which Borda and AHP methods are utilized, to make an extension to the CSSCF. We also make a choice of key technologies for IT industry of Guangdong province, China, in the years 2005 – 2010 by these methods. Generally speaking, in deferent way to synthesize the multiple criteria and opinions the synthesis regulation and results are different. The two methods used in this paper are essentially similar; the results by them to make the choice of key technologies are also basically the same. It needs a further study that whether or not this conclusion applies to other problems.

REFERENCES

[1] Yue Chaoyuan, The Group Decision, Institute of Systems Engineering, Central China Univ. of Tech., 1990
[2] Chen Ting, Decision Analysis, Science Press, Beijing, 1997
[3] Saaty T. L. Decision Making for Leaders: The Analytic Hierarchy Process:for Decision in Complex World, RWS publication, 1990.